CHAPTER TWO

THE ELECTROSTATIC FIELD

Coulomb`s Law:

- The magnitude of the force acting between two electrically charged bodies was studied by Coulomb in 1875.
- He showed that if the bodies were small, compared with their distance apart, then the force F was inversely proportional to the square of their distance apart r

i.e. $F \propto \frac{1}{r^2} \ldots \ldots \ldots \; eqn \; (1).$

- This is known as the inverse law or the square law.

The quantity of charge:

- The S.I. Unit of charge in the coulomb (c), and it is defined as the quantity of charge which passes a section of a conductor in one second, when the current flowing is one ampere.

- By measuring the force F between the charges when their respective magnitudes Q and Q_1 are varied, it will be found that F is proportional to the product QQ_1,

=> $F \propto QQ_1$ -------------eqn (2).

- Combining eqn (1) and eqn (2) => $F \propto \frac{QQ_1}{r^2} \Rightarrow F = \frac{KQQ_1}{r^2}$--- eqn (3) , where K = a constant.

- For certain reasons, k is written as $\frac{1}{4\pi\epsilon_0}$, where ϵ_0 is a constant called the permittivity of free space, if it is assumed that the charges are situated in vacuum,

$$\Rightarrow F = \frac{1}{4\pi\epsilon_0}\frac{QQ1}{r^2}$$

$$\Rightarrow F = \frac{QQ1}{4\pi\epsilon_{0}r^2} --- eqn\ (4).$$

- In this expression, F is measured in Newtons (N), Q in Coulomb (c) and r in metres.

- From equation eqn (4), $\epsilon_0 = \frac{QQ_1}{4\pi F r^2}$

=> the unit of ϵ_0 is $C^2N^{-1}m^{-2}$. Another unit widely used is fared meter^{-1}(FM^{-1}).

- ϵ_0 has a numerical value of 8.854×10^{-12}, and $\frac{1}{4\pi\epsilon_0}$ has an approximate value of 9×10^9.

Permittivity:

- So far we have considered the charges in vacuum, but if the charges are situated in other media such as water, then the force between the charges will be reduced.

-The equation $F = \frac{1}{4\pi\epsilon_0}\frac{QQ1}{r^2}$ is only true in vacuum, and in general we write $F = \frac{1}{4\pi\epsilon}\frac{QQ1}{r^2}$, where ϵ = the permittivity of the medium concerned.

-The permittivity of air at normal pressure is only 1.005 times ϵ_0, i.e of that of vacuum i.e. the permittivity of air = $1.005\epsilon_0$.

- For most purposes, we use the value ϵ_0 for the permittivity of air.

- Since the permittivity of water is about eight times that of vacuum, then the force acting between the charges when they are situated in water, is eight times less than if they were situated the same distance apart in vacuum.

(Q1) Two charges Q_1 and Q_2 are of values 4.0uc and 5.0uc respectively, and are situated in vacuum. If they are 2m apart, calculate the force of repulsion between them. [Take $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$].

Soln:

$Q_1 = 4.0uc = 4 \times 10^{-6}c$.

$Q_2 = 5.0uc = 5 \times 10^{-6}c$ and $r = 2m$.

But since $F = \frac{1}{4\pi\epsilon_0}\frac{Q_1Q_2}{r^2}$

$=> F = \frac{9 \times 10^9 \times 4 \times 10^{-6} \times 5 \times 10^{-6}}{2^2}$

$= \frac{180 \times 10^{-3}}{4} = 45 \times 10^{-3}N$.

(Q2) Calculate the value of two equal charges, if they repel one another with a force of 0.1N when they are situated 50cm apart in vacuum. [Take $\epsilon_0 = 8.9 \times 10^{-12}Fm^{-1}$].

Soln:

Since the charges are equal => $Q = Q_1$, if Q and Q_1 are the two charges.

. Also since $Q = Q_1$, then $F = \frac{1}{4\pi\epsilon_0} . \frac{QQ}{r^2} => F = \frac{1}{4\pi\epsilon_0} . \frac{Q^2}{r^2}$

Since $F = 0.1N$, $r = 50cm = 0.5m$ and $\epsilon_0 = 8.9 \times 10^{-12}$.

$=> 0.1 = \frac{1}{4\pi(8.9 \times 10^{-12})} \times \frac{Q^2}{0.5^2}$

$=> 0.1 = \frac{Q^2}{4 \times 3.14 \times 8.9 \times 10^{-12} \times 0.25)}$

$=> 0.1 = \frac{Q^2}{28 \times 10^{-12})}$

$=> Q^2 = 0.1 \times 28 \times 10^{-12}$,

$=> Q^2 = 2.8 \times 10^{-12}$,

$=> Q = \sqrt{2.8 \times 10^{-12}} = 1.7 \times 10^{-6}$

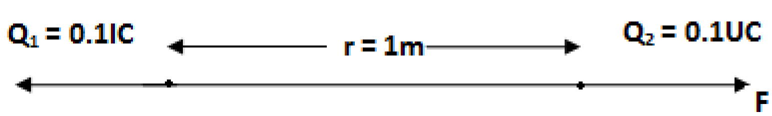
=> value of each charge = $1.7 \times 10^{-6}C$.

N/B: $\frac{1}{4\pi\epsilon_0} = (4\pi\epsilon_0)^{-1}$.

(Q3) The distance between the centres of two small spheres, each of which carries a charge of 0.1uc is 1M. What force does each exert on the other?

[Take $(4\pi \epsilon_0)^{-1} = 9.0 \times 10^9 NM^2C^{-2}$].

Soln:



$Q_1 = 0.1UC = 0.1 \times 10^{-6}$, $Q_2 = 0.1UC = 0.1 \times 10^{-6}C$

$F = \frac{1}{4\pi\epsilon_0} . \frac{Q_1 Q_2}{r^2}$

Since $F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}$

$=> F = (4\pi\epsilon_0)^{-1}\frac{Q_1 Q_2}{r^2}$

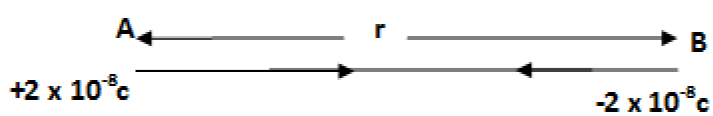
$=> F = \frac{9 \text{ X } 10^9 \text{ X } 0.1 \text{ X } 10^{-6} \text{ X } 0.1 \text{ X } 10^{-6}}{1^2}$

$=> F = 9 \text{ x } 0.1 \text{ x } 0.1 \text{ x } 10^9 \text{ x } 10^{-6} \text{ x } 10^{-6}$,

$=> F = 9.0 \text{ x } 10^{-5}$N.

(Q4) Calculate the electric force between two small charged spheres A and B, 1.0m apart, if they carry charges + 2.0 x 10^{-8}C and -2.0 x 10^{-8}c respectively.[Assume$(4\pi\epsilon_0)^{-1} = 9.0 \text{ x } 10^9 Nm^2C^{-2}$].

Soln:



$Q_1 = 2 \text{ x } 10^{-8C}$ and $Q_2 = -2 \text{ x } 10^{-8}$C

Since $F = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q_1 Q_2}{r^2}$

$=> F = (4\pi\epsilon_0)^{-1}\frac{Q_1 Q_2}{r^2}$

$=> F = \frac{9.0 \text{ X } 10^9 \text{ X } 2.0 \text{ X } 10^{-8} \text{ X } 2.0 \text{ X} 10^{-8}}{1^2}$

$= 3.6 \text{ x } 10^{-6}$N.

N/B:

(i) Since the two charges carry opposite signs, then they will attract each other and for this reason, the arrows shown in the diagram just drawn are directed towards each other.

(ii) In calculation, if a charge carries a negative sign, we drop the negative sign.

(Q4) Determine the force which acts between two charges $Q_1(2 \text{ x } 10^{-6}$C) and $Q_2(15 \text{ x } 10^{-6}$C), if they are situated 3m apart in a medium, whose permittivity is five times that of vacuum. [Take $\frac{1}{4\pi\epsilon_0} = 9 \text{ x } 10^{9)}$].

Soln:

$Q_1 = 2 \times 10^{-6}C$, $Q_2 = 15 \times 10^{-6}C$ and r = 3m.

Let $\in$ = the permittivity of the medium. Since the permittivity of the medium is five times that of vacuum => $\in = 5\in_0$.

Since the permittivity of the medium = $\in$

=> $F = \frac{1}{4\pi\in}\frac{Q_1Q_2}{r^2}$, and since $\in = 5\in_0$,

=> $F = \frac{1}{4\pi(5\in 0)}.\frac{Q_1Q_2}{r^2}$

=> $F = \frac{1}{5(4\pi\in 0)} \times \frac{Q_1Q_2}{r^2}$ =>

$F = \frac{1}{5} \times \frac{1}{4\pi\in_0} \times \frac{2 \times 10^{-6} \times 1.5 \times 10^{-6}}{9} = \frac{1}{5} \times 9 \times 10^9 \times \frac{2 \times .5 \times 10^{-6} \times 10^{-6}}{9} = \frac{9 \times 2 \times 15 \times 10^{-9} \times 10^{-12}}{45}$

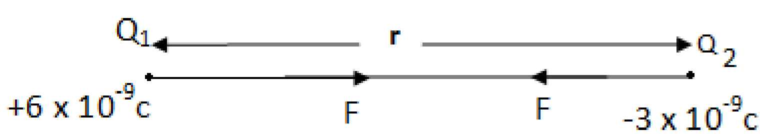
=> $F = 6 \times 10^{-3}N$.

(Q5) (a) With what force do two identical small spheres interact in vacuum, if one carries a charge of $+6.0 \times 10^{-9}c$ and the second, a charge of $-3.0 \times 10^{-9}C$, if the distance between these two spheres is 5.0cm.

(b) With what force will these spheres interact if they are brought into contact and removed to their previous positions.

[Assume $(4\pi \in_0)^{-1} = 9.0 \times 10^9 NM^2C^{-2}$].

Soln:



$Q_1 = 6 \times 10^{-9}c$, $Q_2 = -3 \times 10^{-9}c$, r = 5cm = 0.05m = $5 \times 10^{-2}M$.

Since $F = \frac{1}{4\pi\in_0} . \frac{Q_1Q_2}{r^2}$

=> $F = (4\pi\in_0)^{-1}\frac{Q_1Q_2}{r^2}$,

$$=> F = \frac{9 \text{ x } 10^9 \text{ x } 6.0 \text{ x } 10^{-9} \text{ x } 3.0 \text{ x } 10^{-9}}{(5 \text{ x } 10^{-2})^2}$$

$$= \frac{9 \text{ x } 6 \text{ x } 3 \text{ x } 10^9 \text{ x } 10^{-9} \text{ x } 10^{-9}}{5^2 \text{ x } 10^{-4}} = \frac{162}{25} \text{ x } 10^{-9} \text{ x } 10^4$$

$= > F = 6.5 \text{ x } 10^{-5}. = 65 \text{x } 10^{-1}. \text{ x } 10^{-5}. = 65 \text{x } 10^{-6} = 65UC.$

(a) When the two charges are brought into contact, the positive one which is larger will cancel the effect of the negative charge.

=> the net charge = $Q_1 + Q_2$

= $(6 \text{ x } 10^{-9}) + (-3 \text{ x } 10^{-9})$

= $6 \text{ x } 10^{-9} – 3 \text{ x } 10^{-9} = (6 – 3) 10^{-9}$

= $3 \text{ x } 10^{-9}$.

This charge will be distributed between the two spheres equally,

=> the charge on each sphere = $\frac{3 \text{ x } 10^9}{2}$ = $1.5 \text{ x } 10^{-9}$c.

Since these charges were brought to the previous positions after coming into contact, then the distance between them = 5cm = $(5 \text{ x } 10^{-2})$ m.

Since the two spheres now have identical charges, then F = $\frac{1}{4\pi\epsilon_0} . \frac{QQ}{r^2}$

=> $F = (4\pi\epsilon_0)^{-1}. \frac{Q^2}{r^2}$

$$=> F = \frac{9 \text{ x } 10^9 \text{ x } 1.5 \text{ x } 10^{-9} \text{ x } 1.5 \text{ x } 10^{-9}}{(5 \text{ x } 10^{-2})^2}$$

$$=> F = \frac{9 \text{ x } 1.5 \text{ x } 1.5 \text{ x } 10^9 \text{ x } 10^{-9} \text{ x } 10^{-9}}{5^2 \text{ x } 10^{-4}}$$

$$=> F = \frac{20.3 \, X \, 10^{-9}}{25 \text{ x } 10^{-4}} = 0.8 \text{ x } 10^{-9} \text{ x } 10^4$$

$= 0.8 \text{ x } 10^{-5}$N.

Electric Field:

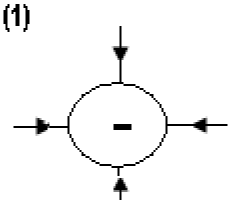
- An electric field is a region where an electric force is experienced.
- The force exerted on a test charge Q, depends of the charge of the body, as well as

the electric field intensity.
- The direction of an electric field is defined by the lines known as electric lines of force or electric flux.

- The electric intensity E at a point may be defined as the flux density at that point, and the flux density is equal at points which are at the same distance from the point charge.

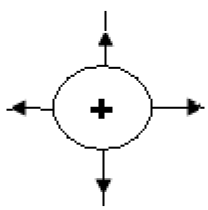
Some lines of forces:

(1)



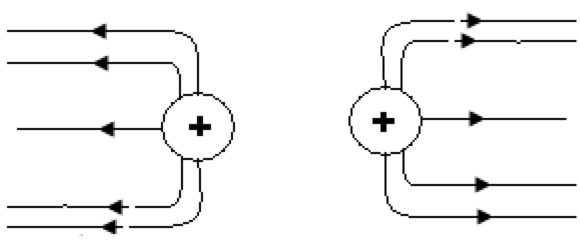
Lines of forces for a negatively charged particle

(2)



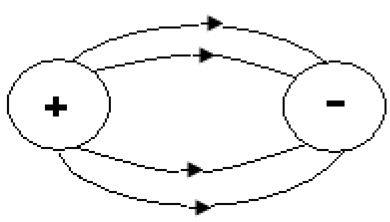
Lines of forces for a positively charged particle.

(3)



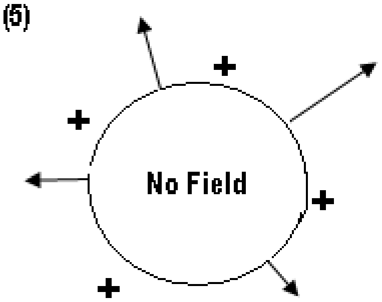
Lines of forces for two like charges

(4)



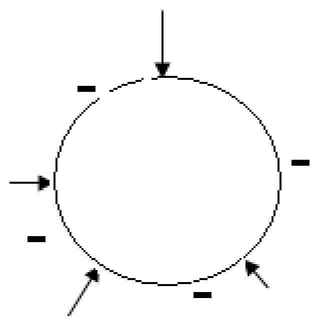
Lines of forces for unlike charges

(5)



Lines of forces of an isolated positively charged sphere.

(6)



Lines of forces of an isolated negatively charged sphere